Chapter 6
Navier-Stokes-alpha model: LES equations with nonlinear dispersion

J. A. Domaradzki

Aerospace and Mechanical Engineering, University of Southern California, Los Angeles, CA 90089-1191, U.S.A.

D.D. Holm

T-Division and CNLS, MS-B284, Los Alamos National Laboratory, Los Alamos, NM 87545, USA

Abstract We present a framework for discussing LES equations with nonlinear dispersion. In this framework, we discuss the properties of the nonlinearly dispersive Navier-Stokes-alpha (NS$-\alpha$) model of incompressible fluid turbulence — also called the viscous Camassa-Holm equations in the literature — in comparison with the corresponding properties of large eddy simulation (LES) equations obtained via the approximate-inverse approach. In this comparison, we identify the spatially filtered NS$-\alpha$ equations with a class of generalized LES similarity models. Applying a certain approximate inverse to this class of LES models restores the Kelvin circulation theorem for the defiltered velocity and shows that the NS$-\alpha$ model describes the dynamics of the defiltered velocity for this class of generalized LES similarity models. We also show that the subgrid scale forces in the NS$-\alpha$ model transform covariantly under Galilean transformations and under a change to a uniformly rotating reference frame. Finally, we discuss in the spectral formulation how the NS$-\alpha$ model retains the local interactions among the large scales; retains the nonlocal sweeping effects of large scales on small scales; yet attenuates the local interactions of the small scales amongst themselves.

6.1 Introduction

The effects of subgrid-scale (SGS) fluid motions occurring below the available resolution of numerical simulations must be modeled. One way of modeling these effects is simply to discard the energy that reaches such subgrid scales. This is clearly unacceptable, though, and many creative alternatives have been offered. A prominent example is the large eddy simulation (LES) approach, see, e.g., [1] – [4]. The LES approach is based on applying a spatial filter to the Navier-Stokes equations. The reduction of flow complexity and information content achieved in the LES approach depends on the characteristics of the filter that one uses, its type and width. In particular, the LES approach introduces a length scale into the description